647510 **BSCalgCC5010** **Seat No: _________**

B.Sc. Semester - 5 (CBCS) Examination

Oct/Nov. - 2018

MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 (A) Answer the following. (Any one) (07)

(1) If (X, d) is a metric space, then show that $d_1 : X \times X \to \mathbb{R}$; $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ is a bounded metric on X.

(2) Let X be a metric space and $A, B \subset X$. Show that $\overline{A \cup B} = \overline{A \cup B}$.

(B) Attempt any one. (04)

(1) Prove that if (X, d) is a metric space, then $|d(x, z) - d(y, z)| \leq d(x, y), \forall x, y, z \in X$.

(2) Prove that: Any finite subset of a metric space is closed.

(C) Attempt any one. (03)

(1) Define following terms.

(i) Discrete metric space. (ii) Limit point of a set. (iii) Open set.

(2) Prove that: In a metric space arbitrary union of open sets is an open set.

Que-2 (A) Answers the following (Any one). (07)

(1) Show that : $\lim_{n \to \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \ldots + \frac{1}{2n}\right] = \frac{\pi}{4}$

(2) Show that : $f(x) = 2 - 3x$ is integrable on [1, 3] and $\int_1^3 (2 - 3x)dx = -8$.

(B) Attempt any one. (04)

(1) If $f : [a, b] \to \mathbb{R}$ is a bounded function and $P, P' \in$ P $[a, b]$ be such that $P \in P'$, then show that $L(P, f) \leq L(P', f)$.

(2) If $f \in R[a, b]$, then show that.

$$M(b - a) \leq \int_a^b f(x)\, dx \leq M(b - a).$$

Where m and M are the infimum and supermum of f on $[a, b]$, respectively.

(C) Attempt any one. (03)

(1) Let $f : [0, 1] \to \mathbb{R}$ be a function defined as

$$f(x) = \begin{cases} \frac{1}{5} & x \text{ is rational} \\ \frac{1}{2} & x \text{ is irrational} \end{cases}$$

Show that f is not R-integrable.

(2) Let $f : [a, b] \to \mathbb{R}$ is a bounded and $P \in P[a, b]$, then show that $L(P, -f) = - U(P, f)$.

Que-3 (A) Answers the following (Any one) (07)

(1) State and Prove: First Mean Value Theorem of Integral Calculus.

(2) Show that $GL(2, \mathbb{R})$ is a non-abelian group under matrix multiplication.

(B) Attempt any one. (04)

(1) Express $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{2r}{n} + 3\right)$ as definite integral.

(2) If $P_1, P_2 \in P\,[a, b]$, then show that $L(P_2, f) \leq U(P_1, f)$

(C) Attempt any one. (03)

(1) Let $G = \{ f_{a,b} \mid f_{a,b} : \mathbb{R} \to \mathbb{R},\ f_{a,b}(x) = ax + b\ (x \in \mathbb{R}),\ a \neq 0,\ a, b \in \mathbb{R} \}$ be a group under composition of mappings. Find the identity element of G and inverse of $f_{2,3}$.

(2) Find the remainder obtained on dividing 5^{355} by 16.

Que-4 (A) Answers the following. (Any one). (07)

(1) State and prove Cayley's Theorem.

(2) Let G be a group and $H \leq G$. For $a, b \in G$ define $a \equiv b \pmod{H} \Leftrightarrow a^{-1} b \in H$. Prove that $\equiv$ is an equivalence relation on G. Also find $\mathrm{cl}(a)$ where $a \in G$.

(B) Attempt any one. (04)

(1) Let G be a group. If $a^2 = e, \forall a \in G$, then show that G is abelian.

(2) Let G be a group, $H < G$ and $a, b \in G$. Show that either $aH = bH$ or $aH \cap bH =$.

(C) Attempt any one (03)

(1) In usual notation prove that A_n is a normal subgroup of S_n.

(2) Let $f: G \to \bar{G}$ be a group isomorphism. If $\bar{N} \lhd \bar{G}$, then show that $f^{-1}(\bar{N}) \lhd G$.

Que-5 (A) Answer the following (Any one). (07)

(1) State and prove Euler's Theorem.

(2) If G is a group such that $(ab)^n = a^n b^n$, for three consecutive integers n, then show that G is abelian.

(B) Attempt any one. (04)

(1) Let H and K be finite sub groups of a group G. If $O(H)$ and $O(K)$ are relatively prime, then prove that $H \cap K = \{e\}$.

(2) Prove that a group of prime order is cyclic.

(C) Attempt any one. (03)

(1) Using the Fermat's Theorem, show that if p is an odd prime then

$$1^p + 2^p + \cdots + (p-1)^p \equiv 0 \pmod{p}$$

(2) Let G be a group and $a \in G$. Show that $O(a^{-1}) = O(a)$.

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